

“What Display Number?”

Contest entry by Joseph K. Horn

INTRODUCTION

The Question

When photographing HP calculators, what is the “best” number to have in their displays?

The Answer

There are many excellent answers to that question, but deciding which one of them is the “best” depends on which of the following criteria are most important to you:

- Use the fewest possible keystrokes.
- Show as many different digits (0 thru 9) as possible.
- Omit as few HP models as possible.
- It is a collection of high-end, low-end, scientific, or business models.

The Meta-Answer

This paper and its accompanying “**Best Number Table**” present a precise method for selecting the one “best display number” based on your precise criteria. It therefore serves as a “meta-answer,” that is, an answer that generates answers.

Two Notes of Almost-Despair

Note #1: The original contest write-up specified that the number should be obtainable in two keystrokes, or at most three. Unfortunately, the Contest Update made that goal impossible to achieve, because it specified that we are “trying to photograph a large group of machines – say 25 or more different machines.” Different models require a different number of keystrokes to perform the same function. *There simply does not exist any two- or three-keystroke sequence that always yields all ten digits (0 thru 9) on 25 different HP calculators, unless that collection is specified ahead of time (see bullet #4 in The Answer above).*

Note #2: The write-up specified that the number should contain as many digits (0 thru 9) as possible. It is obvious that this quest will run into an inverse ratio between the number of keystrokes and the number of missing digits. *But the write-up failed to specify the desired ratio.*

These two notes originally tempted me to despair, but instead I have resorted to the intrepid approach taken by this paper, namely, to offer a method by which anybody can find the one best number for their particular photographic needs.

THE BEST NUMBER TABLE

The following is to be read with a copy of the table in hand.

Column 1: **HP Model**

Hewlett Packard calculator model number. The letters following the model numbers are HP's actual model name, except for “a” which means “the original model didn't have any letter.” All such letters are in lower-case here, although HP used a mixture of upper- and lowercase.

This particular list of 85 models is taken from a recent HP newsletter. It omits the models that were never commercially available, or were in the “palmtop” series (95LX etc), or didn't run on batteries. It also omits the HP-12C Prestige due to its being identical to the 12C Platinum.

Column 2: **FIX, SCI digits**

This shows the number of digits that each calculator is able to display. This information is essential for figuring out how many digits 0 thru 9 are missing from each prospective “best display number.” Each column entry is of the form “NN, SS+EE”. NN is the number of mantissa digits in full FIX mode (or STD or Norm or All mode). SS is the number of mantissa digits in full SCI or ENG mode. EE is the number of digits in the exponent in SCI or ENG mode.

Example: The entry for the HP-01 is “8, 4+2”. This means that the HP-01 ordinarily shows numbers using 8 digits, but numbers in scientific notation are shown with 4 mantissa digits with 2 exponent digits.

Exceptions:

“10 only” – The HP-10a always displays 10 digits; it has no scientific notation mode.

“12, 10\1” – The 12 means a 12-digit mantissa like usual, but the 10\1 means that the display of scientific-notation numbers is variable. When the exponent is 1 digit long, then 10 mantissa digits are displayed. As the exponent gets longer (with a negative sign counting as a digit place), the mantissa shrinks accordingly. Therefore, “12, 10\1” means “12, 10+1” and “12, 9+2” and “12, 8+3” and “12, 7+4”.

“12, 12\1” is the unique and unfortunate display design of the HP 35s, which requires the user to scroll the display to see the last digits of long numbers. If the entire number happens to fit on the screen without scrolling, then it fits a “12, 12\1” format.

Column 3: Logic

As all RPN fans know, algebraic-expression calculators often require more keystrokes than RPN to get a job done. For example, $\text{TAN}(7)$ can be done in 2 keystrokes in RPN (namely, [7] [TAN]) but it takes more than 2 keystrokes on an algebraic-expression calculator (e.g. [TAN] [7] [=] or [TAN] [7] [ENTER] or [TAN] [7] [)] [ENTER] or [T] [A] [N] [(] [7] [)] [ENTER]). The keystroke counts in the rest of the **Best Number Table** are based on this fact.

Notation:

“alg” means ordinary algebraic logic, with algebraic order of operations (e.g. $1+2*3$ yields 7, not 9). This is NOT “algebraic expression” logic. $\text{TAN}(7)+1$ is calculated by pressing [7] [TAN] [+] [1] [=].

“exp” means algebraic expression logic, sometimes also called “command-line interpreter” or CLI logic. It performs no calculations until the [ENTER] key is pressed (sometimes called [=] or [EXE] or other things). $\text{TAN}(7)+1$ is calculated by pressing [TAN] [7] [+] [1] [ENTER]. Some machines require the user to explicitly close all open parentheses.

“admac” means adding-machine logic. This is unique to the HP-10a. Its [+] key doubles as its [=] key. It is the most primitive kind of algebraic logic possible. It does not follow algebraic order of operations.

“bus” means business-calculator logic. This is the same as “alg” except that it does not follow algebraic order of operations (e.g. $1+2*3$ yields 9, not 7).

“RPN” means Reverse Polish Notation. $\text{TAN}(7)+1$ is calculated by pressing [7] [TAN] [1] [+].

“RPNbus” means that the model has both “RPN” and “bus” logic, selectable by the user.

“RPNalg” means that the model has both “RPN” and “alg” logic, selectable by the user.

“RPN+2” means that the model has all three logic types, “RPN” and “alg” and “bus”, selectable by the user.

“RPL” means reverse-Polish LISP. This is unique to the 48/49/50 series. It is the infinite-stack version of RPN. $\text{TAN}(7)+1$ is calculated the same way with RPL and RPN.

“CALC” means CALC MODE. It is unique to the HP-71B. It is an “active command-line interpreter” which doesn’t wait until [ENTER] is pressed, but as the user types the expression, all subexpressions that are finished are immediately calculated, and replaced with the intermediate results. $\text{TAN}(7)+1$ is calculated by pressing [TAN] [7] [)] [+] [1] [ENTER].

Column 4: $\frac{1}{x}$

The 1/x function is a single-keystroke function on only half of the models. On the other models, it is either a shifted function (thus requiring 2 keystrokes) or an algebraic-expression function (thus requiring [ENTER] to be pressed) or both (3 keystrokes). Some algebraic models have no 1/x function at all, in which case the 3-keystroke sequence [1] [/] [=] can be used as its functional equivalent. This column gives the exact number of keystrokes required to calculate 1/x on every calculator model.

The last entry at the bottom of this column is the average number of keystrokes required by 1/x across all the models in the entire **Best Number Table**, namely, 1.91 keystrokes.

Column 5: $\frac{1}{81} = 0.012345679$

The first proposed “best display number” is 0.012345679 which can be calculated with 3 keystrokes on the HP 50g. However, it requires more than 3 keystrokes on some models. And it is missing the digit 8 on all models, regardless of display mode. (It is also missing 6 and 9 on the HP-01). This column shows the exact number of required keystrokes for each model, as well as the missing digits. The average keystrokes across all the models is shown at the bottom of the **Best Number Table** (page 3), and below that is the average number of missing digits. Every model can calculate this number.

Column 6: $\frac{1}{81} - 11 = -10.987654321$

The second proposed “best display number” is -10.987654321 which can be calculated with 6 keystrokes on the HP 50g. Although that’s a lot of keystrokes, this number has the unique property of being do-able on every model AND missing no digits (except on the HP-01 which MUST miss at least 2 digits because its mantissa is only 8 digits long!) AND the shortest keystroke sequence to have both of those properties. As an added bonus, it even includes a negative sign. The usual averages and totals appear at the bottom of the **Best Number Table**.

Column 7: **EEX**

Two proposed “best display numbers” use the EEX key (called E or EXP on some models). Unfortunately, this is a shifted key on roughly one third of all HP models. And there is no EEX key at all on six models! This column shows how many keystrokes EEX takes on each model, with the average shown at the bottom (zeros omitted).

Column 8: $\frac{1}{81E6} = 1.23456790E-8$

The third proposed “best display number” is 1.2345679E-8 which has no missing digits and a bonus negative sign. It requires fewer keystrokes than the previous proposed number. It has the drawback of not being do-able on the six models that have no EEX key.

Column 9: **TAN**

The next proposed number uses the TAN function. This column shows how many keystrokes each model needs to calculate a tangent. TAN is a primary key on less than half of the models, and is buried in a menu on two models. Notice that 20 models have no TAN function at all.

Column 10: **tan(7)** in DEG mode = **.122784560903**

The fourth proposed “best display number” is .122784560903 which has no missing digits IF all twelve digits are shown. If only ten digits can be shown, then the missing “3” can be “faked” by using ENG display mode (122.7845609 E-03). No matter what you do, at least one digit (0 thru 9) will be missing on 24 models. It’s not even a do-able number on the 20 models that have no TAN function.

Column 11: LN

The last proposed number uses the LN function. This column shows how many keystrokes each model requires to perform LN. Seven models do not have the LN function at all. It is a primary-key function on only 15 models.

Column 12: ln(1E9) = 2.07232658369E1

The fifth and final proposed “best display number” is 20.7232658369 which can be done on more models than the previous proposed number, but takes more keystrokes and has more missing digits. As in all things, it’s a trade-off.

CONCLUSION

The “best display number” for a photograph of a group of calculators can be easily determined from the **Best Number Table**, depending on which models you have, and which criteria are most important to you.

Example: If you have a collection of scientific (not business) models, and you consider the number of keystrokes to be more important than the number of missing digits, then **TAN(7)** (in DEG mode!) is the “best display number” for you. However, if you have a collection of almost every model, and you want every digit showing, then **1/81-11** is the “best display number”. An excellent compromise is **1/81** which takes few keystrokes, misses only one digit, and can be done on all models. One of the five “best display numbers” will best fit any given criteria.

Historical note: I started with manual brute-force searching. When that grew tedious, HP 50g programs and UBASIC programs were used to search for function results with the minimum missing digits 0 thru 9 in different display modes and rounded to various lengths. The model-specific data in the **Best Number Table** were painstakingly deduced from the ASCII-art representations of all the HP calculator keyboards in Craig Finseth’s online “HP Database.”

Kudos to Richard Nelson for this delightful challenge!

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